DC Lines Modeling Details

Power flows along AC lines are completely governed by physics. The AC flows in the network are fixed with a given set of power injections and a defined network topology. DC lines, however, are fully controllable. Aurora's nodal solution finds the optimal flows on all active DC lines while ensuring the lowest system production cost. Broadly speaking, flows on the controllable elements are modeled as paired injections at the terminal buses. Constraints are added to ensure that flows are always within specified upper and lower control limits. Relaxation of flow limits are allowed at the same cost as the global Branch and Corridor Shadow Price Cap. Losses on DC ties are based on the input loss rate and are accounted on the receiving terminal bus.

The following section presents the mathematical description of DC line logic using an example.

Mathematical Formulation

Consider a DC line connecting two buses named B_1 and B_2 with a loss rate specified as L. Further, B_1 has generation, denote as G_1. B_1 and B_2 are connected to the rest of the network via one or more AC lines. Please note that this example extends to all DC line setup without loss of any generality.

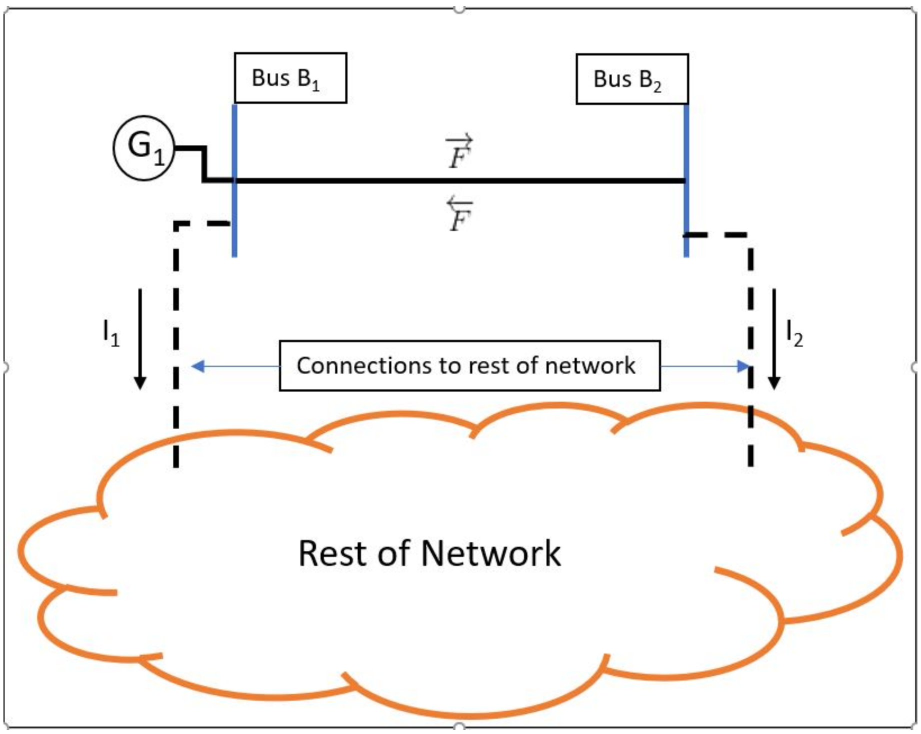

Figure 1: DC Tie setup between Buses B_1 and B_2.

As shown in Figure 1, the flow on the DC lines in the direction of from B_1 to B_2 is denoted as $\overrightarrow{F}$, while $\overleftarrow{F}$ represents the flow from B_2 to B_1.

The following constraints are added to SCED LP[1]

1. **Power Injection at B_1:** $I_1 = G_1 - \overrightarrow{F} + (1 - L)\overleftarrow{F}$
2. **Power Injection at B_2:** $I_2 = (1 - L)\overrightarrow{F} - \overleftarrow{F}$
3. **Limits on DC flows:** $LowerLimit \leq \overrightarrow{F} - \overleftarrow{F} \leq UpperLimit$

[1] Similar constraints are also added to the SCUC MILP if using Commitment Optimization.

Constraints 1 and 2 capture the power injections at the two terminals, while Constraint 3 ensures that the optimal solution is within the specified bounds. In essence, the flows on DC lines are viewed as paired injections and withdrawal at the terminal buses. Losses on the DC ties are accounted at the receiving terminal.

Aurora performs additional checks to ensure that $\overleftarrow{F}$ and $\overrightarrow{F}$ are not simultaneously non-zero in the final optimal solution.

Injection Variables I_1 and I_2

Variables I_1 and I_2 capture the amount of power injected to the AC network at bus B_1 and B_2 respectively. If Solution Method = Angles, the variables are utilized in network power flow equations. See documentation on Angles method for further details. If Solution Method = Shift Factor, I_1 and I_2 are included in the system power balance equation as well as for the determination of branch and corridor flows from the PTDF values.

Notes

- Island structures in the network are determined based on the AC lines. Two sets of buses that are connected by DC ties, but are not otherwise connected via any AC lines, will be identified as two different islands. See the discussion of "Allow Multiple Islands in Network" switch for details on how Aurora handles islands in the network.
- Multi-circuit DC lines are allowed. Constraints 1 and 2 are modified to allow flow on each active circuit.
- Wheeling rates between Loadflow Areas or Aggregate Areas are applied to flows on DC ties as well.